

Reprezentarea grafica a functilor reale

Pentru a trasa graficul unei functii , parcurgem mai multe etape :

1) Domeniul maxim de definitie

- a) gasirea domeniului maxim de definitie
- b) $G_f \cap O_x \Rightarrow f(x)=0$
- c) $G_f \cap O_y \Rightarrow x=0$, $f(x)=$ o valoare
- d) $\lim_{x \rightarrow \pm\infty} f(x)$ (daca e constanta $\Rightarrow y=k \Rightarrow$ asimptota orizontala)

2) Semnul functiei

- a) semnul functiei
- b) paritatea functiei
 - $f(x)=f(-x) \Rightarrow$ functia e simetrica fata de axa O_y
 - $f(x)=-f(x) \Rightarrow$ functia e simetrica fata de origine
- c) continuitatea functiei
- d) periodicitatea

3) Asimptote

- a) orizontale
- b) verticale
- c) oblice

4) Derivata intai

- a) calculul derivatei intai
- b) radacinile derivatei intai si valorile functiei pe radacinile derivatei
- c) tabelul

5) Derivata a doua

- a) calculul derivatei a doua

- b) radacinile derivatei a doua si valorile functiei pe radacinile derivatei
 - determinarea punctelor de inflexiune , de maxim si minim local
- c) semnul derivatei a doua

6) Tabelul de variatie al functiei

| | |
|--------|--|
| X | |
| F'(x) | |
| F''(x) | |
| F(x) | |

7) Trasarea graficului

- in grafic se incepe cu trasarea asimptotelor

Exemple:

$$f(x) = x^4 - 8x^2$$

1) a) $f: \mathbf{R} \rightarrow \mathbf{R}$

b) $f(x) = 0$

$$x^4 - 8x^2 = 0$$

$$x^2(x^2 - 8) = 0$$

$$x_1 = x_2 = 0$$

$$x_3 = 2\sqrt{2}$$

$$x_4 = -2\sqrt{2}$$

c) $f(0) = 0 - 0 = 0$

d) $\lim_{x \rightarrow \pm\infty} f(x) = \pm\infty$

2)

a)

| | | | |
|-----------|--------------|---|-------------|
| x | $-2\sqrt{2}$ | 0 | $2\sqrt{2}$ |
| x^2 | +++++0+++++ | | |
| $x^2 - 8$ | +++0----- | 0 | -----0+++ |
| f(x) | +++0----- | 0 | -----0+++ |

b) $f(x) = f(-x)$

=> functie para
=> graficul este simetric fata de axa Ox

3) Asimptote nu exista

4) Derivata intai

$$f'(x) = 4x^3 - 16x = 4x(x^2 - 4)$$

$$x_1 = 0$$

$$x_{2,3} = \pm 2$$

$$f(0) = 0$$

$$f(2) = -16$$

$$f(-2) = -16$$

| | | | | | |
|-----------|-------------------------|----|---|---|-----------|
| x | $-\infty$ | -2 | 0 | 2 | $+\infty$ |
| x | -----0+++++ | | | | |
| $x^2 - 4$ | +++++0-----0+++++ | | | | |
| f'(x) | -----0+++++0-----0+++++ | | | | |

5) Derivata a doua

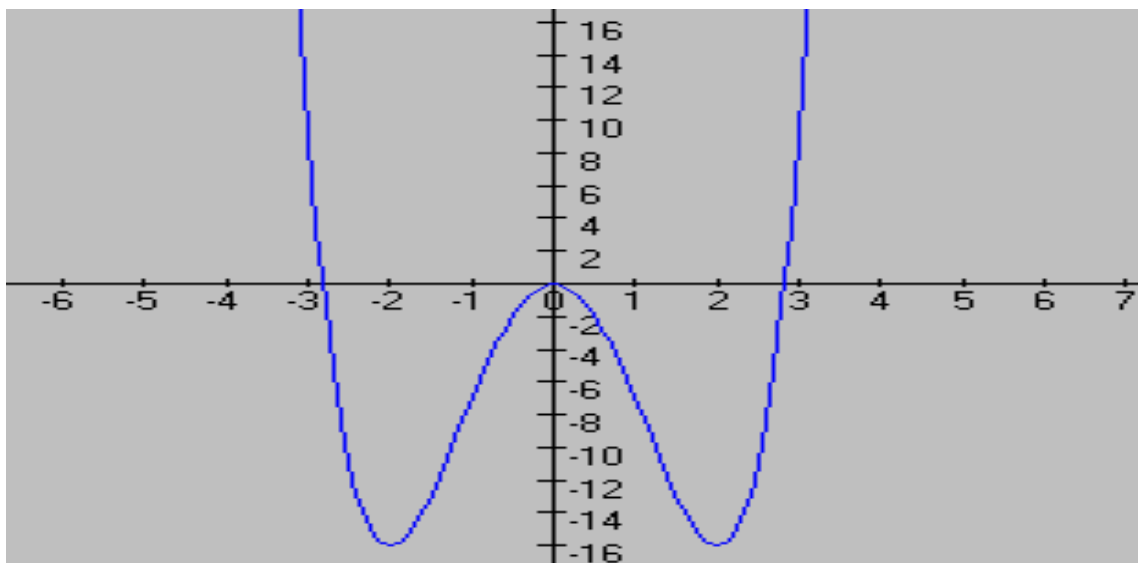
$$f''(x) = 12x^2 - 16$$

$$x_{1,2} = \pm \frac{2\sqrt{3}}{3}$$

$$f\left(\frac{2\sqrt{3}}{3}\right) = -\frac{80}{9}$$

$$f\left(-\frac{2\sqrt{3}}{3}\right) = -\frac{80}{9}$$

| | | | | | | | | | |
|--------|-------------------------|--------------|---------|------------------------|--------|-----------------------|---------|-------------|-----------------|
| X | $-\infty$ | $-2\sqrt{2}$ | -2 | $-\frac{2\sqrt{3}}{3}$ | 0 | $\frac{2\sqrt{3}}{3}$ | 2 | $2\sqrt{2}$ | $+\infty$ |
| f'(x) | -----0+++++0-----0+++++ | | | | | | | | |
| f''(x) | +++++0-----0+++++ | | | | | | | | |
| f(x) | $-\infty$ | -----0 | -----16 | +++ $-\frac{80}{9}$ | -----0 | +++++ $-\frac{80}{9}$ | -----16 | +++++0 | +++++ $+\infty$ |
| | | | m | i | M | i | m | | |



$$f(x) = \frac{x^2 - 16}{x}$$

1) a) $f: \mathbf{R} \setminus \{0\} \rightarrow \mathbf{R}$

b) $f(x) = 0$

$$x^4 - 16 = 0$$

$$x_1 = 4 \quad A(4, 0) \quad ; \quad B(-4, 0)$$

$$x_2 = -4$$

c) $f(0) = \text{nu exista}$

d) $\lim_{x \rightarrow \pm\infty} f(x) = \pm\infty$

2)

a)

| | | | | | |
|------------|-----------|-------|-------|-------|-----------|
| x | $-\infty$ | -4 | 0 | 4 | $+\infty$ |
| $x^2 - 16$ | +++++ | 0 | ----- | 0 | +++++ |
| x | ----- | ----- | 0 | +++++ | +++++ |
| f(x) | ----- | 0 | +++++ | ----- | 0 |

$$x \in (-\infty, -4) \cup (0, 4) \Rightarrow f(x) < 0$$

$$x \in (-4, 0) \cup (4, +\infty) \Rightarrow f(x) > 0$$

b) $f(x) = f(-x)$

=> functie para

=> graficul este simetric fata de axa Ox

c) functia este continua pe $\mathbf{R} \setminus \{0\}$

3) Asimptote

$y=x \Rightarrow$ asimptota oblica la $\pm\infty$

$$\lim_{x \rightarrow 0} f(x) = \frac{x^2 - 16}{x} = \frac{-16}{0^+} = \infty \Rightarrow x=0 \text{ asimptota verticala la } \pm\infty$$

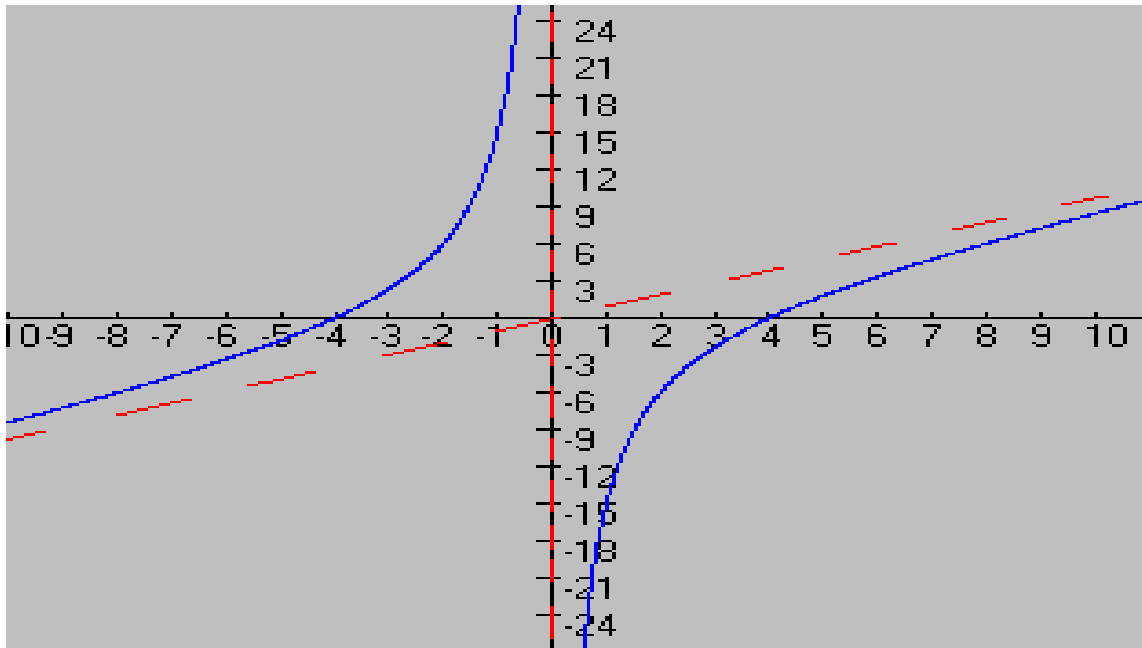
4) Derivata intai

$$f'(x) = \frac{x^2 + 16}{x^2} > 0$$

5) Derivata a doua

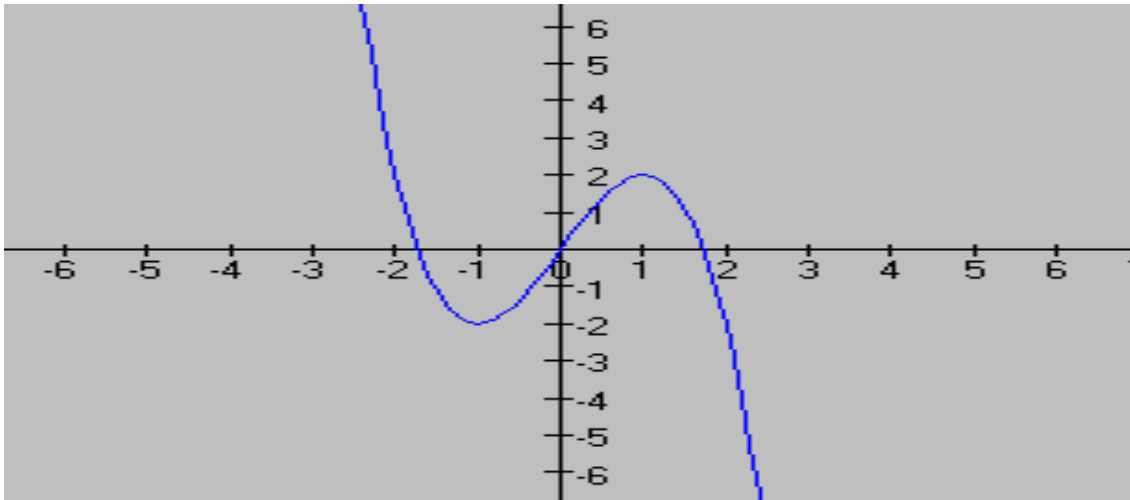
$$f''(x) = \frac{x - 32}{x^3}$$

| | | | |
|----------|-----------|-------|-----------|
| X | $-\infty$ | 0 | $+\infty$ |
| $f'(x)$ | +++++ | +++++ | +++++ |
| $f''(x)$ | +++++ | ----- | ----- |
| $f(x)$ | $-\infty$ | | $+\infty$ |

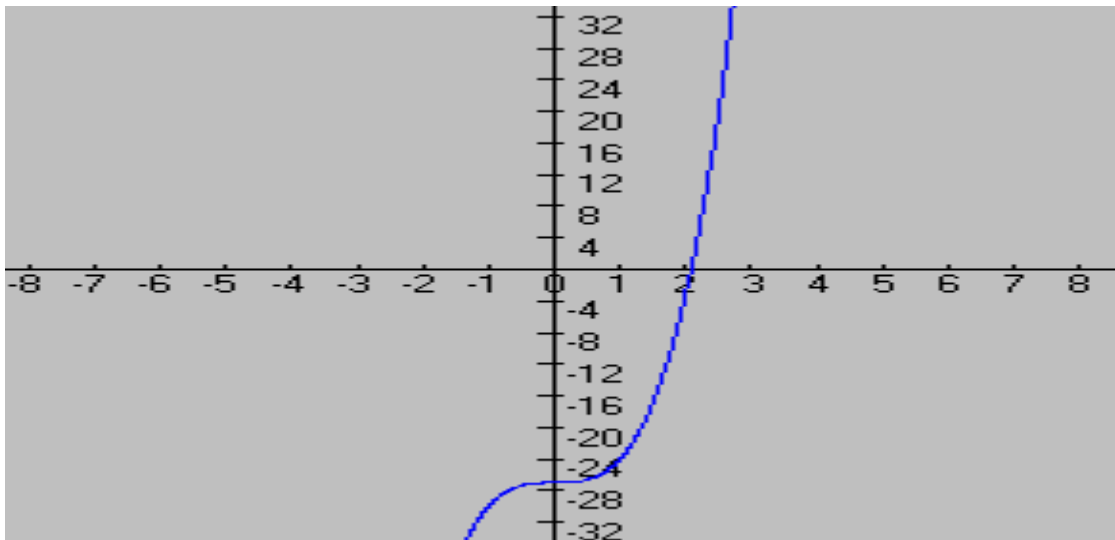


Alte grafice de functii :

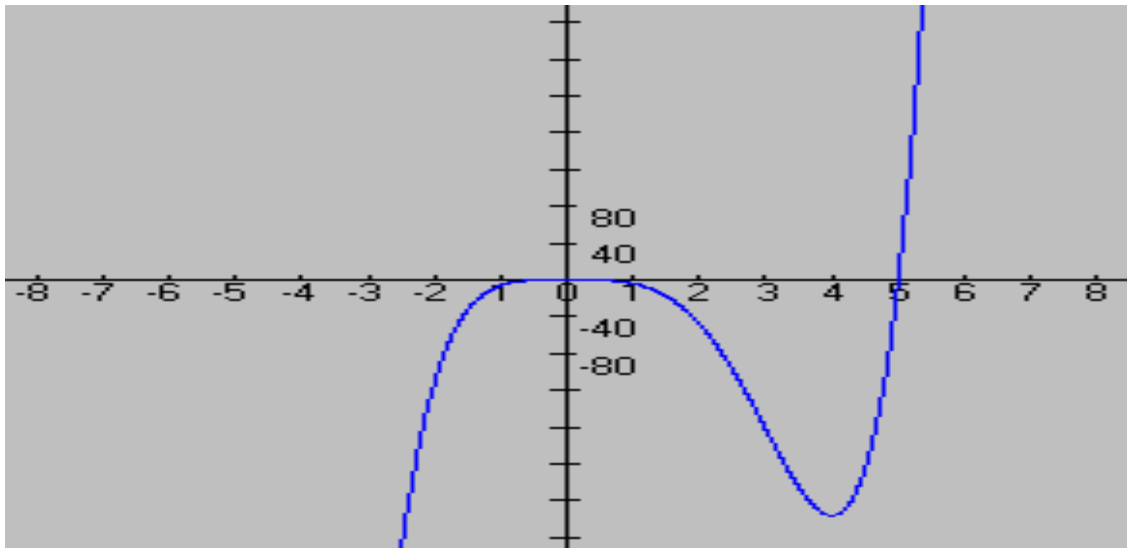
$$1) f(x) = -x^3 + 3x$$



$$2) f(x) = 3x^3 - 27$$



$$3) f(x) = x^5 - 5x^4$$



$$4) f(x) = \frac{2x}{x^2 + 1}$$

